

QCD phase transition with a power law chameleon scalar field in the bulk

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Abstract

In this work, a brane world model with a perfect fluid on brane and a scalar field on bulk has been used to study quark-hadron phase transition. The bulk scalar field has an interaction with brane matter. This interaction comes into non-conservation relation which describe an energy transfer between bulk and brane. Since quark-hadron transition curly depends on the form of evolution equations therefore modification of energy conservation equation and Friedmann equation comes into some interesting results about the time of transition. The evolution of physical quantities relevant to quantitative of early times namely energy density ρ temperature T and scale factor a have been considered utilizing two formalisms as crossover formalism and first order phase transition formalism. The results show that the quark-hadron phase transition in occurred about nanosecond after big bang and the general behavior temperature is similar in both of two formalism.

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1 Introduction

Recently, models of the Universe with extra dimensions have attracted serious attention. Among the most well-known of these models are the original brane-world scenarios considered by Randall and Sundrum in 1999 [1, 2]. These scenarios present an interesting picture of the Universe in which the standard matter fields are confined to a four-dimensional hypersurface (the brane) embedded in higher-dimensional space-time (the bulk). The graviton, by exception, is allowed to propagate in the bulk. In this model the four-dimensional Friedmann equation can be recovered in low energy limit with a positive tension for brane embedded in five-dimensional Anti-de Sitter space-time. On the other hand, the model suggests that the dynamic of the early Universe (high energy regime) should be modified related to the contribution of quadratic term of energy density and bulk Weyl tensor. This result is so important because we have some changes in the basic equations which describe the cosmological and also the astrophysical dynamics of the Universe, and this has been recently the subject of so many research works [3, 4, 5, 6, 7].

Drawing on motivations from particle physics, there has been some recent interest in the possibility that non-gravitational matter-energy can be exchanged between the brane and bulk [8, 9]. In this case, the evolution of the matter fields on the brane is modified by

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the introduction of new terms in the energy-momentum tensor which describes the matter-energy exchange between the brane and the bulk. In Ref. [10], it was found that such a model can explain the present cosmic acceleration as a consequence of the flow of matter from the bulk to the brane. Interestingly, the model also seems to predict the observed suppression of the lowest multipole moments in the power spectrum of the cosmic microwave background. Some features of the brane-world with bulk-brane matter-energy transfer have already been studied. For example, the cosmological evolution of a brane with a chameleon scalar field in the bulk was studied in Ref. [11, 12], the cosmological evolution of a brane with a general matter content in the bulk was studied in Ref. [10], and the reheating of a brane-world Universe with bulk-brane matter-energy transfer was studied in Refs. [13]. Another suggestion which leads to some interesting models even in standard model of cosmology is scalar field. The problem of scalar field in brane-world scenario attracted great interests [14, 15, 16]. According to string theory, it is also possible that the brane-world models contain bulk scalar field which is free to propagate in extra dimensions [17]. This scalar field also could interact with matter and tension of brane which is called dilaton scalar field [17, 18, 19]. It is natural and attractor to study the effect bulk scalar field on the brane evolution and it is not unexpected that these kinds of models represent some interesting consequences. Before that it is remarkable to mention that there is such a scalar field in standard cosmology with non-minimal coupling to matter which called chameleon scalar field. Based on that, we called the bulk scalar field as a chameleon scalar field [20, 21]. The model also comes to some absorbing results about the brane evolution. In [22], it is shown that although bulk is free of matter but due to the interaction to matter, the scalar field takes a mass which depends on the brane energy density. This fact comes to this results that the mass of scalar field on four-dimensional brane is not small and therefore correction to the Newton law cannot be large because of propagation of scalar field in the bulk. There can be some other appealing feature for this model in reheating area as well, refer to [22]. In this area, the scalar field decay to other particles. The matter scalar field, with zero bare mass, will find an effective mass as $m_\psi = g\phi(t)$ which in this model m_ψ is much smaller than m_ϕ and so according to [23] this plays the crucial role in the reheating process. In addition, the model is able to predict and accelerated expansion in late time and an exponential expansion in the early times which could be related to inflationary epoch of the universe. The interesting point about this kind of scalar field is that the conservation relation is modified (in addition to the Friedmann equation) and the model possesses this ability to suggest the energy-exchange term. Then, there is no need to recommend a manual term. The energy-exchange is directly resulted from the introduced term in the action which describes the interaction between matter and scalar field, and this can be one of the advantages of the model. Based on above mentioned feature for this model of scalar field in braneworld scenario, and its compatibility with string theory suggestion it is found out interesting to study the ability of such a models in other era of universe evolution.

On the other hand, according to the standard model of cosmology, as the Universe expanded and cooled it passed through a series of symmetry-breaking phase transitions which might have generated topological defects [24]. This early Universe phase transition could have been of first, second, or higher order, and it has been studied in detail for over three decades [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45]. We note that the possibility of no phase transition was considered in Ref. [46, 47, 48, 49, 50]. The phase transition using both first order formalism and crossover formalism has been considered in standard cosmology [51, 52, 53, 54] with viscosity effect [55, 56, 57] that causes a kind of non-conservation relation. Furthermore, the first order and the crossover

approaches to phase transitions were studied in the brane-world [24, 58], DGP brane-world [59], and in Brans-Dicke models of brane-world gravity [60] without any bulk-brane energy transfer.

In the present paper, we study the quark-hadron phase transition (or *QCD phase transition*) in a brane-world with a chameleon scalar field in the bulk. We find that, in this model, the non-minimal coupling between the scalar field and matter modifies the energy conservation equation for the matter fields on the brane. This fact, together with the Friedmann equation in the brane-world scenario (which is different from the Friedmann equation in the four-dimensional case), causes an increased expansion in early times and this has an important effect on the cosmological phase transition. The model is a generalized form of some other models [24, 60, 61] which have considered phase transition and their works have been published in some valid journals such as *Phys. Rev. D*, *Nucl. Phys. B*, *Class. Quant. Grav.*. Moreover, the following reasons lead one to this result that our model possesses this suitability to be considered

- The model is motivated from extra dimensions models which have a special place in the physicist research works.
- The model includes a bulk scalar field based on the suggestion of string/M theory.
- The presence of bulk-brane energy-exchange term in the model is based on the prediction of particle physics.
- Avoiding freely selection of energy-exchange term.

This paper is organized as follows. In Sec. 2, we introduce the model and derive the equations of motion. Sec. 2 is related to the study of the QCD phase transition (smooth crossover approach), and in Sec. 3. We review the first-order phase transition and consider it in our model in Sec. 5 we summarize our results.

2 General Framework

In accordance with the brane-world scenario, we treat our usual four-dimensional space-time (the brane) as an embedded submanifold of a five-dimensional spacetime (the bulk). In order to accommodate the chameleon scalar field ϕ in the bulk, we postulate the following action

$$A = \int d^5x \sqrt{-g} \left\{ \frac{1}{2} R^{(5)} - \Lambda_5 - \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right\} - \int d^4x \sqrt{-h} \mathcal{L}_4. \quad (1)$$

We have expressed the action in terms of the five-dimensional coordinates (x^0, x^1, x^2, x^3, y) on the bulk, such that the brane is the hypersurface described by $y = 0$. Moreover, we have assumed units in which the five-dimensional Planck mass equals one. The signature of the five-dimensional metric $g_{\mu\nu}$ is $-++++$. The first term corresponds to the Einstein-Hilbert action generalized to five dimensions together with a scalar field ϕ (the chameleon scalar field) whose potential is given by the function $V(\phi)$, which is assumed to be almost flat. Note that, due to the dimensional units of the energy-momentum tensor, the scalar field no longer has dimension M , but rather M^2 . Also, the dimension of the potential is M^5 . The determinant of $g_{\mu\nu}$ and the five-dimensional Ricci scalar are denoted by g and $R^{(5)}$ respectively. Λ_5 , in the first integral, is the bulk cosmological constant. The second term describes the coupling between the scalar field and the matter fields. The metric $g_{\mu\nu}$ in the

bulk induces a metric $h_{\mu\nu}$ on the embedded four-dimensional brane. The determinant of $h_{\mu\nu}$ is denoted as h . \mathcal{L}_4 , in the brane action, is defined as $\mathcal{L}_4 = L_m(\psi_m, \tilde{h}_{\mu\nu}) - \lambda$ where the matter field is ψ_m and the matter Lagrangian on the brane is denoted by L_m , and λ denotes brane tension. We assume that the non-minimal coupling between the scalar field and the matter field is given by

$$\tilde{h}_{\mu\nu} := S(\phi)h_{\mu\nu}, \quad (2)$$

where $S(\phi)$ is an analytical function of the scalar field.

By varying the action (1) with respect to $g_{\mu\nu}$, one obtains the five-dimensional Einstein field equation

$${}^5G_{\mu\nu} = T_{\mu\nu}^{(\Lambda)} + T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{(b)}, \quad (3)$$

where $T_{\mu\nu}^{(\Lambda)}$ and $T_{\mu\nu}^{(\phi)}$ respectively denote the bulk cosmological constant energy-momentum tensor and the total energy-momentum tensor for the scalar field, which is defined as

$$T^{(\Lambda)\mu}_{\nu} = (-\Lambda_5, \Lambda_5, \Lambda_5, \Lambda_5, \Lambda_5) \quad (4)$$

and

$$T_{\mu\nu}^{(\phi)} = \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \left\{ \frac{1}{2} (\nabla \phi)^2 + V(\phi) \right\}. \quad (5)$$

$T_{\mu\nu}^{(b)}$ is the brane energy-momentum tensor which is given by

$$l5aT_{\mu\nu}^{(b)} = \frac{2}{\sqrt{-h}} \frac{\partial (\sqrt{-h}\mathcal{L}_4)}{\partial h^{\mu\nu}} \delta(y). \quad (6)$$

In fact, one can write

$$l6aT^{(b)\mu}_{\nu} = \delta(y) \text{diag}(-\rho_b, p_b, p_b, p_b, 0). \quad (7)$$

Here, ρ_b and p_b denote the brane energy density and pressure respectively. These quantities include the tension λ of the brane, so that

$$\begin{aligned} \rho_b &= \rho_m + \lambda \\ p_b &= p_m - \lambda, \end{aligned} \quad (8)$$

where ρ_m and p_m denote the matter density and pressure respectively; see Refs. [62, 63, 64]. Varying the action (1) with respect to ϕ leads to the field equation for the scalar field

$${}^5\Box\phi = V_{,\phi} + \frac{1}{2} \frac{\tilde{S}(\phi)}{S} T^{(b)} \delta(y), \quad (9)$$

where ${}^5\Box$ is the five-dimensional d'Alembertian, $\tilde{S}(\phi) := dS(\phi)/d\phi$ and $T^{(b)}$ is the trace of brane energy momentum tensor.

We introduce a Friedmann-Lemaître-Robertson-Walker (FLRW) metric with a maximally symmetric 3-geometry γ_{ij}

$$ds^2 = -n^2(t, y)dt^2 + a^2(t, y)\gamma_{ij}dx^i dx^j + dy^2. \quad (10)$$

Our brane is the hypersurface given by $y = 0$, and we also take Z_2 symmetry into account. It should be mentioned that the metric coefficients are continuous but their first derivatives with respect to y are discontinuous, and furthermore their second derivatives with respect to y include the Dirac delta function. Substituting the above metric into the field equations, one gets that the non-vanishing components of Einstein tensor are

$$l9a^{(5)}G_{00} = -\frac{3}{a^2}\left\{-\dot{a}^2 + a'^2n^2 + aa''n^2\right\}, \quad (11)$$

$$^{(5)}G_{ij} = \frac{\delta_{ij}}{n^3}\left\{2a\ddot{a}n + 2\dot{n}a\dot{a} + 2n^2n'aa' - n\dot{a}^2 + a'^2n^3 + 2aa''n^3 + n^2a^2n''\right\}, \quad (12)$$

$$^{(5)}G_{05} = -\frac{3}{an}\left\{\dot{a}'n - n'\dot{a}\right\}, \quad (13)$$

$$^{(5)}G_{55} = \frac{3}{a^2n^3}\left\{-a\ddot{a}n + \dot{n}a\dot{a} + n^2n'aa' - n\dot{a}^2 + a'^2n^3\right\}. \quad (14)$$

Here, dots denote derivatives with respect to time and primes denote derivatives with respect to the fifth coordinate. Since the second derivative of the metric involves the Dirac delta function, we write, following Ref. [62, 63]

$$l13a'' = \hat{a}'' + [a']\delta(y), \quad (15)$$

where \hat{a}'' is the non-distributional part of the double derivative of $a(t, y)$, and $[a']$ is the jump in the first derivative across $y = 0$, which can be expressed schematically as

$$l14[a'] = a'(0^+) - a'(0^-). \quad (16)$$

The junction functions can be obtained by matching the Dirac delta functions in the components of Einstein tensor with the components of the brane energy-momentum tensor. From the $(0, 0)$ and (i, j) components of field equations, we have, respectively

$$\frac{[a']}{a_0} = -\frac{1}{3}\rho_b, \quad (17)$$

$$\frac{[n']}{n_0} = \frac{1}{3}(2\rho_b + 3p_b), \quad (18)$$

where the subscript “0” means that we evaluate on $y = 0$. These equations are the same as the junction relations in Ref. [62, 63]. On the other hand, according to Eq. (9), we get

$$\frac{\ddot{\phi}}{n^2} - \phi'' + \left(\frac{3\dot{a}}{an^2} - \frac{\dot{n}}{n^3}\right)\dot{\phi} - \left(\frac{n'}{n} + \frac{3a'}{a}\right)\phi' = -V_{,\phi}(\phi) - \frac{1}{2}\frac{\tilde{S}(\phi)}{S}T^{(b)}\delta(y). \quad (19)$$

Matching the Dirac delta functions on both sides of Eq. (19) gives, for $y = 0$:

$$[\phi'_0] = \frac{1}{2}\frac{\tilde{S}(\phi_0)}{S(\phi_0)}T^{(b)}. \quad (20)$$

Also, because we have Z_2 symmetry, one can obtain a'_0 , n'_0 and ϕ'_0 functions from the junction conditions.

The $(0, 5)$ component of the Einstein tensor reads:

$$^5G_{05} = T_{05}^{(\phi)} = \dot{\phi}\phi',$$

($T^{(b)05} = 0$). Now, by substituting \dot{a}' and n' from (17) and (??), and using Z_2 symmetry, one can obtain generalized continuity equation on the brane

$$\dot{\rho}_b + 3H(\rho_b + p_b) = 2\dot{\phi}_0\phi'_0. \quad (21)$$

This equation is the modified continuity relation of energy density and indicates a bulk-brane energy transfer. Note that we have set $n_0 = 1$ in all relations of this section.

From the (5, 5) component of the Einstein tensor, one gets the second order (or generalized) Friedmann equation

$$\frac{\ddot{a}_0}{a_0} + \frac{\dot{a}_0^2}{a_0^2} = -\frac{1}{36}\rho_b(\rho_b + 3p_b) - \frac{\Lambda_5}{3} - \frac{1}{3}\left[\frac{\dot{\phi}_0^2}{2} + \frac{\phi_0'^2}{2} - V(\phi_0)\right]. \quad (22)$$

The first Friedmann equation on the brane is obtained from the (0, 0) component of the field equation

$$H^2 = \left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{1}{36}\rho_b^2 + \frac{\Lambda_5}{3} + \frac{1}{3}\left[\frac{\dot{\phi}_0^2}{2} + \frac{\phi_0'^2}{2} + V(\phi_0)\right]. \quad (23)$$

Here we have assumed that the non-distributional part of the double derivative of $a(t, y)$ with respect to the fifth coordinate, \hat{a}_0'' , is zero. It is seen that equation (23) agrees with the results obtained in Refs. [65] and [66] for brane world cosmology and is completely different from the standard cosmological model because in standard cosmology, $H \propto \sqrt{\rho}$ rather than ρ . It is seen that in Eqs. (19), (21) and (23) there is an additional free parameter.

To determine behavior of scalar field, one should solve scalar equation (19). A kind of interaction between matter and scalar field which has been assumed in the model brings scalar field dependence for brane matter density [11, 12]. Hence finding out the exact solution of equation encounter difficulties. Therefore in these cases and to simplify the problem, a specific function for scalar field is supposed. In accordance with Ref. [67, 68], we shall assume that the scalar field can be expressed as a power of the scale factor

$$l22\phi_0 = N_0\left[\frac{a(t)}{a(t_0)}\right]^\xi = Na^\xi. \quad (24)$$

Here, $a(t_0)$ is the scale factor at the present time, $a(t_0) = 1$, N , and ξ are constant. Using Eqs. (8), (20), and (??) we can rewrite Eqs. (23), (19) and (21) as (by imposing Randall-Sundrum fine-tuning, namely $\Lambda_4 = \frac{1}{12}\lambda^2 + \Lambda_5 = 0$)

$$H = \frac{1}{\sqrt{6(6 - (\xi\phi_0)^2)}} \left[\rho_m(2\lambda + \rho_m) + \frac{3}{8} \left(\frac{\tilde{S}(\phi_0)}{S(\phi_0)} \right)^2 T^{(b)2} + 12V(\phi_0) \right]^{1/2} \quad (25)$$

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 = \frac{1}{24} \frac{\tilde{S}(\phi_0)}{S(\phi_0)} T^{(b)2} - V_{,\phi}(\phi_0), l24 \quad (26)$$

$$\dot{\rho}_b + 3H(\rho_b + p_b) = \frac{\xi HT^{(b)}}{2} \left[\frac{\tilde{S}(\phi_0)}{S(\phi_0)} \phi_0 \right], l25 \quad (27)$$

In order to gain better insight, we choose $S(\phi_0) = s_0\phi^m$. For this choice, Eqs. (25), (??)

and (??) are rewritten as

$$H = \frac{1}{\sqrt{6(6 - (\xi\phi_0)^2)}} \left[\rho_m(2\lambda + \rho_m) + \frac{3}{8} \left(\frac{mT^{(b)}}{\phi_0} \right)^2 + 12V(\phi_0) \right]^{1/2} \quad (28)$$

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 = \frac{1}{24} \frac{m}{\phi_0} T^{(b)2} - V_{,\phi}(\phi_0), \quad (29)$$

$$\dot{\rho}_b + 3H(\rho_b + p_b) = \frac{m}{2} \xi H T^{(b)}. \quad (30)$$

A specific function for scalar field potential density should be assumed to continue our study in subsequent sections. Two scalar field potential energy densities, exponential and inverse power law, are commonly used in discussion of the chameleon mechanism. Here we consider the inverse power law potential energy density [69, 70]

$$V(\Phi) = M^5 \left(\frac{M^2}{\phi} \right)^\alpha, \quad (31)$$

where $\alpha > 0$, M is a constant mass scale and the scalar field, ϕ , has M^2 dimension. The authors of Refs. [20, 21, 71] consider the solar system constraints for a model with this potential and find out that for small values of $\alpha \in (0, 2)$ the magnitude of M is $\sim 10^{-3}$ eV. Therefore, the potential may be written as

$$V(\phi) \sim \frac{10^{-3(2\alpha+5)}}{\phi^\alpha} \text{ eV}^5. \quad (32)$$

Since the characteristic energy density scales of other quantities such as ρ , p , λ , and constants of the model, are of order a MeV, the scalar field potential energy density term is very small compared to other terms in the Lagrangian density and we can ignore it.

3 QCD Phase Transition

In this section, we are going to examine the physical quantities related to the quark-hadron phase transition. The results will be applied in the context of a brane-world scenario with a chameleon scalar field in the bulk. The phase transition in QCD can be characterized by the truly singular behavior of the partition function, and may be either a first or second order phase transition. It can only be a crossover with rapid changes in observable that strongly depend on the values of the quark masses.

To study the quark-hadron phase transition, we need equation of state for matter in both quark and hadron phase regimes. There exist various procedures for finding equations of state (EoS). In most of the recent calculations for 2+1 flavor QCD, EoS has been estimated. The most extensive calculations of EoS have been performed with fermion formulation on lattice with temporal extent $N_t = 4, 6$ [72, 73, 74, 75], $N_t = 8$ [76] and $N_t = 6, 8, 10$ [77]. In the high temperature region, $T > 250$ MeV, the trace anomaly can be calculated accurately, so one can use the lattice data for the trace anomaly in this region in order to construct a realistic equation of state. In the low temperature region, the trace anomaly is affected by a large discretization effect, but the hadronic resonance gas (HRG) model is well-suited for building a realistic equation of state in the low temperature region [78].

Since in brane world scenario, all standard particles are confined on a four-dimensional hypersurface, then equation of state for matter in brane world scenario is the same as equation

of state in the standard four-dimensional model of cosmology. In fact the equation of state comes from the matter energy-momentum tensor, and presence of extra dimension in the model has no effects on this tensor. There are some other works which the authors consider quark-hadron phase transition in brane world scenario [24, 58, 60] and they have applied the same equation of state in the four dimensional in their works.

3.1 High temperature region

As noted above, one can use the lattice data for the trace anomaly in order to find the equation of state in the high temperature region, $T > 250$ MeV [78]. In this regime, radiation-like behavior can be seen, and the data can be fit to a simple equation of state which reads as follows

$$l29\rho(T) \approx \alpha_r T^4, \quad (33)$$

$$p(T) \approx \sigma_r T^4, l30 \quad (34)$$

where $\alpha_r = 14.9702 \pm 009997$ and $\sigma_r = 4.99115 \pm 004474$.

Substituting Eqs. (??) and (??) into (??) we obtain

$$l31H = -\frac{4\alpha_r T^3 \dot{T}}{A_0 T^4 + A_1}, \quad (35)$$

where

$$A_0 = 3(\alpha_r + \sigma_r) - \frac{m}{2}\xi(3\sigma_r - \alpha_r)$$

$$A_1 = 2m\xi\lambda.$$

By integrating (??) we have

$$l32a(T) = a_c \left(A_0 T^4 + A_1 \right)^{-\alpha_r/A_0}, \quad (36)$$

where a_c is a constant of integration, and from the modified Friedmann equation (28), and (??) the time derivative of temperature, \dot{T} , is

$$l33\dot{T} = -\frac{A_0 T^4 + A_1}{4\alpha_r T^3 \sqrt{6(6 - (\xi\phi)^2)}} \left\{ \alpha_r T^4 (2\lambda + \alpha_r T^4) + \frac{3m^2}{8\phi^2} [(3\sigma_r - \alpha_r)T^4 - 4\lambda]^2 \right\}^{1/2} \quad (37)$$

this expression describe the behavior of temperature as a function of time in the brane world universe with chameleon scale field in the bulk in the quark phase. The transition region in the crossover regime can be defined as the temperature interval $250 \text{ MeV} < T < 700 \text{ MeV}$. We have plotted the numerical results of Eq. (??) in Fig. 1 for $\lambda = 10^9 \text{ MeV}^4$ in the interval $250 \text{ MeV} < T < 700 \text{ MeV}$. The plot shows two curve for Eq.(??) related to two initial value of temperature as $T = 700 \text{ MeV}$ and $T = 250 \text{ MeV}$, and the interval space between this two curves is shaded. This figure shows the effective temperature of the Universe as a function of cosmic time in the quark-gluon phase (QGP) according to the crossover procedure, which is obtained from lattice QCD data. This figure tell us by passing the time the Universe becomes cooler and the QGP is occurred (finished) at bout $0.2 - 0.3$ nanosecond after the big bang.

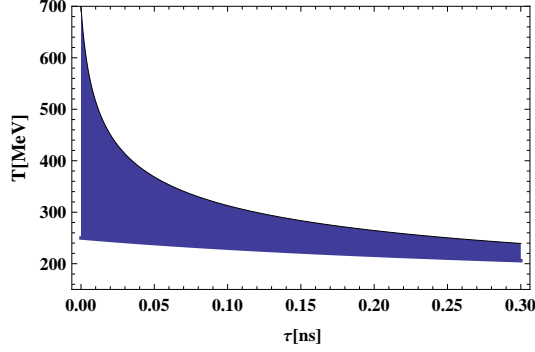


Figure 1: T versus τ according to high temperature region of the smooth crossover procedure in the brane gravity with chameleon scalar field in the bulk for $\lambda = 10^9 \text{ MeV}^4$. We have set $N = 0.2 \times 10^3$, $\xi = 0.0002$, and $m = 1$.

3.2 Low temperature region

As mentioned above, the hadronic resonance gas (HRG) model yields a realistic equation of state in the low temperature regime, $T \lesssim 180 \text{ MeV}$ [78]. In the HRG scenario, the confinement phase of QCD is treated as a non-interacting gas of fermions and bosons [79, 80]. The idea of the HRG model is to implicitly account for the strong interaction in the confinement phase by looking at the hadronic resonances only, since these are basically the only relevant degrees of freedom in that phase. The HRG result can be parameterized for the trace anomaly [78]

$$l34 \frac{I(T)}{T^4} \equiv \frac{\rho - 3p}{T^4} = a_1 T + a_2 T^3 + a_3 T^4 + a_4 T^{10}, \quad (38)$$

where $a_1 = 4.654 \text{ GeV}^{-1}$, $a_2 = -879 \text{ GeV}^{-3}$, $a_3 = 8081 \text{ GeV}^{-4}$, $a_4 = -7039000 \text{ GeV}^{-10}$. In lattice QCD, through the computation of the trace anomaly $I(T) = \rho(T) - 3p(T)$, one can estimate the pressure, energy density, and entropy density, with the help of the usual thermodynamic identities. The pressure difference at two temperatures T and T_{low} can be expressed as the integral of the trace anomaly

$$l35 \frac{p(T)}{T^4} - \frac{p(T_{\text{low}})}{T_{\text{low}}^4} = \int_{T_{\text{low}}}^T \frac{dT'}{T'^5} I(T'). \quad (39)$$

For sufficiently small values of the lower integration limit, $p(T_{\text{low}})$ can be neglected due to the exponential suppression [81]. The energy density $\rho(T) = I(T) + 3p(T)$ can then be calculated. Using Eqs. (??) and (??) we obtain

$$l36 \rho(T) = 3a_0 T^4 + 4a_1 T^5 + 2a_2 T^7 + \frac{7a_3}{4} T^8 + \frac{13a_4}{10} T^{14}, \quad (40)$$

$$l37 p(T) = a_0 T^4 + a_1 T^5 + \frac{a_2}{3} T^7 + \frac{a_3}{4} T^8 + \frac{a_4}{10} T^{14}, \quad (41)$$

where $a_0 = -0.112$. In this case we consider the era before the phase transition (quark-gluon phase) at low temperatures, in which the Universe is in the confinement phase and can be treated as a non-interacting gas of fermions and bosons [79, 80]. From the conservation

Eq.(??) we have

$$l38H = -\frac{2[12a_0T^3 + 20a_1T^4 + B_0(T)]\dot{T}}{6[4a_0T^4 + 5a_1T^5 + B_1(T)] + m\xi B_2(T)}, \quad (42)$$

where

$$l39B_0(T) = 14a_2T^6 + 14a_3T^7 + \frac{91}{5}a_4T^{13}, \quad (43)$$

$$B_1(T) = \frac{7}{3}a_2T^7 + 2a_3T^8 + \frac{7}{5}a_4T^{14}, l40 \quad (44)$$

$$B_2(T) = a_1T^5 + a_2T^7 + a_3T^8 + a_4T^{14} + 4\lambda.l41 \quad (45)$$

To obtain the scale factor as a function of temperature we should integrate Eq.(??). To solve the Eq. (??) we need to make some approximating assumptions. Whereas the coefficient $a_2 = -879 * 10^{-9} \text{MeV}^{-3}$, $a_3 = 8081 * 10^{-12} \text{MeV}^{-4}$ and $a_4 = -7039 * 10^{-27} \text{MeV}^{-10}$, are very smaller than a_0 and a_1 , then for obtaining the scale factor, we neglect those terms which the coefficients of them are a_2, a_3, a_4 . So by this approximation, one can obtain the scale factor as

$$l46a(T) \cong a_c U_1^{-12a_0/(30+m\xi)a_1} \exp \left\{ -U_0 \text{Arctan} \left[\frac{(30+m\xi)a_1T + 12a_0}{\sqrt{m\xi(30+m\xi)a_1\lambda - 36a_0^2}} \right] \right\}, \quad (46)$$

where

$$U_1 = \left[\left(5 + \frac{m\xi}{6} \right) a_1 T^2 + 4a_0 T + \frac{2m\xi}{3} \lambda \right], \quad (47)$$

$$U_0 = \left[\frac{8(12 + 3m\xi)a_0}{(30 + m\xi)\sqrt{m\xi(30 + m\xi)a_1\lambda - 36a_0^2}} \right]. \quad (48)$$

Eqs. (28), (??), (??) and (??), the time derivative of temperature is

$$\begin{aligned} l51\dot{T} &= \frac{-\left[6\left(4a_0T^4 + 5a_1T^5 + B_1(T) \right) - m\xi B_2(T) \right]}{2\sqrt{6(6 - (N\xi a^\xi)^2)}\left(12a_0T^3 + 20a_1T^4 + B_0(T) \right)} \\ &\times \left\{ \rho_m(2\lambda + \rho_m) + \frac{3m^2}{8\phi_0^2}(3p - \rho - 4\lambda)^2 \right\}^{\frac{1}{2}}. \end{aligned} \quad (49)$$

This expression describes the behavior of temperature as a function of cosmic time in the QGP for the brane gravity with chameleon scale field in the bulk. In fact this relation shows the era before phase transition at low temperature where the universe is in the confinement phase and is treated as a non-interacting gas of fermions and bosons.

We solved Eq. (??) numerically and the result is plotted in Fig. 2 for $\lambda = 10^9 \text{ MeV}^4$ in interval $30 \text{ MeV} < T < 180 \text{ MeV}$. The plot shows two curve for Eq.(??) related to two initial value of temperature as $T = 180 \text{ MeV}$ and $T = 30 \text{ MeV}$, and the interval space between this two curves is shaded. This figure shows the behavior of the temperature as a function of cosmic time τ . From this figure one can find that for low temperature region, $30 \leq T \leq 180$, the QGP of the Universe is occurred (finished) at about 15-30 nanosecond after the big bang. We see that QGP in the low temperature region in smooth crossover approach takes place after high temperature region, and this agrees with lattice QCD data analysis prediction.

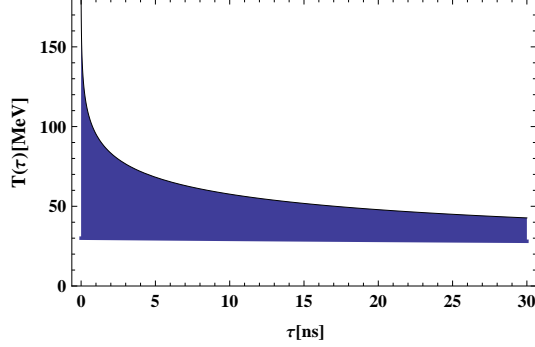


Figure 2: T versus τ according to the low temperature region of the smooth crossover procedure in the brane gravity with chameleon scalar field in the bulk for $\lambda = 10^9 \text{ MeV}^4$ in the low region. We have set $N = 0.2 \times 10^3$, $\xi = 0.002$, and $m = 1$.

4 First order quark-hadron phase transition

Whereas discretization effects in lattice calculations could be large, the HRG model in the intermediate temperature region $180 \text{ MeV} \lesssim T \lesssim 250 \text{ MeV}$ is no longer reliable and our study in the previous section shows this issue. Therefore the evolution of the Universe in the interval $180 \leq T < 250$ should to be studied again using a non-crossover approach.

On the other hand, as mentioned in Section 3, the quark-hadron phase transition in QCD is characterized by the singular behavior of the partition function, and may be the first or second order phase transition [43]. In this section we assume that the quark-hadron phase transition is first order and occurs in the intermediate temperature region $T \lesssim 250 \text{ MeV}$ too. Accordingly, we shall take the equation of state for matter in the quark-gluon phase to be given by [43]:

$$l52\rho_q = 3a_q T^4 + U(T), \quad p_q = a_q T^4 - U(T). \quad (50)$$

Here, the subscript q denotes quark-gluon matter and $a_q = 61.75(\pi^2/90)$. The potential energy density $U(T)$ is [43]:

$$l53U(T) = B + \gamma_T T^2 - \alpha_T T^4, \quad (51)$$

where B is the bag pressure constant, $\alpha_T = 7\pi^2/20$, and $\gamma_T = m_s^2/4$, where m_s , the mass of the strange quark, is in the $60 - 200 \text{ MeV}$ range. This form of U comes from a model in which the quark fields interact with a chiral field formed by the π meson field together with a scalar field. Results obtained from low energy hadron spectroscopy, heavy ion collisions, and from phenomenological fits of light hadron properties give a value of $B^{1/4}$ between 100 and 200 MeV [24].

In the hadron phase, one takes the cosmological fluid to be an ideal gas of massless pions and nucleons described by the Maxwell-Boltzmann distribution function with energy density ρ_h and pressure p_h . Hence, the equation of state in the hadron phase is

$$l54p_h = \frac{1}{3}\rho_h = a_\pi T^4, \quad (52)$$

where $a_\pi = 17.25(\pi^2/90)$.

The critical temperature T_c is defined by the condition $p_q(T_c) = p_h(T_c)$ [76]. Taking $m_s = B^{1/4} = 200$ MeV, the critical temperature is

$$l55T_c = \left[\frac{\gamma_T + \sqrt{\gamma_T^2 + 4B(a_q + \alpha_T - a_\pi)}}{2(a_q + \alpha_T - a_\pi)} \right]^{\frac{1}{2}} \approx 125 \text{ MeV}. \quad (53)$$

Since the phase transition is first order, all physical quantities, such as the energy density, pressure, and entropy, exhibit discontinuities across the critical curve.

4.1 Behavior of temperature in the QGP for general $U(T)$

In order to investigate the quark-hadron phase transition in this section, we shall first obtain the scale factor by using the conservation relation and an especial equation of state, as in the previous section. Using Eqs. (??), (??), and the conservation relation (??), we get

$$l56H = -\frac{\left[2(3a_q - \alpha_T)T^3 + \gamma_T T\right]\dot{T}}{(6a_q - m\xi\alpha_T)T^4 + m\xi\gamma_T T^2 + m\xi(B + \lambda)}. \quad (54)$$

By integrating Eq. (??), the scale factor can be acquired as a function of temperature. Namely

$$l57a(T) = a_c \left[U_0 T^4 + m\xi\gamma_T T^2 + m\xi(B + \lambda) \right]^{\frac{a_T - 3a_q}{2U_0}} \times \exp \left\{ \frac{U_1}{2U_0 U_2} \text{Arctan} \left[\frac{2U_0 T^2 + m\xi\gamma_T}{U_2} \right] \right\}, \quad (55)$$

where,

$$U_0 = 4(3a_q - \alpha_T), \quad (56)$$

$$U_1 = -6\gamma_T a_q (2 - m\xi), \quad (57)$$

$$U_2 = \left[4m\xi(B + \lambda)(6a_q - m\xi\alpha_T) - m^2\xi^2\gamma_T^2 \right]^{1/2}. \quad (58)$$

Therefore, the Freidmann equation and the conservation relation lead us to following relation for \dot{T}

$$l58\dot{T} = -\frac{(6a_q - m\xi\alpha_T)T^4 + m\xi\gamma_T T^2 + m\xi(B - \lambda)}{\sqrt{6(6 - (\xi\phi_0)^2)} \left[2(3a_q - \alpha_T)T^3 + \gamma_T T \right]} \times \left\{ \rho_q(T) \left(2\lambda + \rho_q(T) \right) + \lambda \left(\frac{m}{\phi} \right)^2 (\alpha_T T^4 - \gamma_T T^2 - B - \lambda)^2 \right\}^{\frac{1}{2}} \quad (59)$$

where $\rho_m(T) = (3a_q - \alpha_T)T^4 + \gamma_T T^2 + B$. We have plotted the numerical results of Eq. (??) in Fig. 3 for $\lambda = 10^9$ MeV⁴(solid line), $\lambda = 5 \times 10^9$ MeV⁴(Dashed), $\lambda = 10 \times 10^9$ MeV⁴(Dotted line), $\lambda = 15 \times 10^9$ MeV⁴(Dashed-dotted line). This figure shows that the effective temperature of the Universe in the quark-gluon phase (QGP) decreases with passing time and reaches to the critical temperature at about (0.05- 0.2) nanosecond after the big bang. Moreover by increasing the brane tension the decreasing of the temperature will be faster.

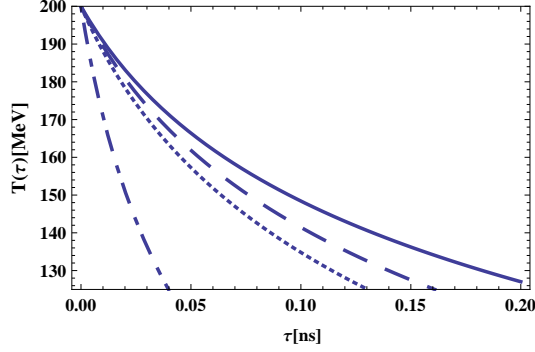


Figure 3: T versus τ in the quark-gluon phase for $\lambda = 10^9 \text{ MeV}^4$ (solid line), $\lambda = 5 \times 10^9 \text{ MeV}^4$ (Dashed), $\lambda = 10 \times 10^9 \text{ MeV}^4$ (Dotted line), $\lambda = 15 \times 10^9 \text{ MeV}^4$ (Dashed-dotted line) in the brane world gravity with chameleon scalar field in the bulk. We have set $N = 0.2 \times 10^4$, $\xi = 0.002$, and $m=1$.

4.2 Behavior of temperature in the QGP for $U(T) = B$

One popular model that deals with quark confinement, is that of an elastic bag which allows the quarks to move around freely, and the potential energy density is constant. In this case, the equation of state for the quark matter is given by the bag model, namely $p_q = (\rho_q - 4B)/3$. Therefore, using the conservation relation, we get that

$$l59 H = -\frac{6a_q T^3 \dot{T}}{6a_q T^4 + C_1}, \quad (60)$$

in which

$$C_1 = m\xi(B + \lambda).$$

This first order differential equation provides us with the scale factor function

$$l60 a(T) = a_c \left(6a_q T^4 + C_1 \right)^{-1/4} \quad (61)$$

Finally, using the Friedmann equation, we find that the time evolution of the temperature T is described as follows

$$l61 \dot{T} = -\frac{(6a_q T^4 + C_1)}{6a_q T^3 \sqrt{6(6 - (\xi\phi_0)^2)}} \times \left\{ \left(3a_q T^4 + B \right) \left(2\lambda + 3a_q T^4 + B \right) + 6\frac{m^2}{\phi^2} (B + \lambda)^2 \right\}^{1/2} \quad (62)$$

We have plotted the numerical results of Eq. (62) in Fig. 4 for $\lambda = 10^9 \text{ MeV}^4$ (solid line), $\lambda = 5 \times 10^9 \text{ MeV}^4$ (Dashed), $\lambda = 10 \times 10^9 \text{ MeV}^4$ (Dotted line), $\lambda = 15 \times 10^9 \text{ MeV}^4$ (Dashed-dotted line). In this case, $U(T) = B$, the decreasing of temperature is slower than the general case of $U(T)$. This figure shows that the effective temperature of the Universe decreases with passing time in the QGP at about (0.1-0.2) nanosecond after the big bang. One can see that the QGP in the high temperature region in the smooth crossover approach is very similar with the QGP in the first order phase transition formalism but QGP in the low temperature region of smooth crossover approach has taken place later with respect to high temperature region of crossover approach and first order phase transition formalism.

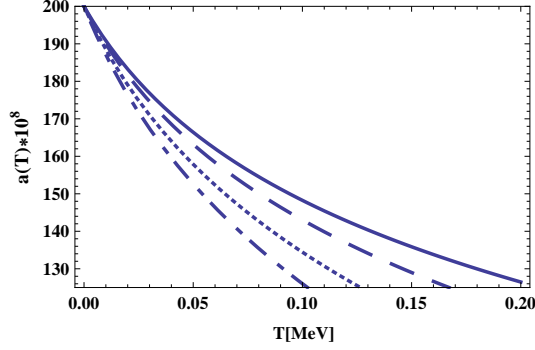


Figure 4: T versus τ in the quark-gluon phase for $\lambda = 10^9 \text{ MeV}^4$ (solid line), $\lambda = 5 \times 10^9 \text{ MeV}^4$ (Dashed), $\lambda = 10 \times 10^9 \text{ MeV}^4$ (Dotted line), $\lambda = 15 \times 10^9 \text{ MeV}^4$ (Dashed-dotted line) in the brane world gravity with chameleon scalar field in the bulk. We have set $N = 0.2 \times 10^4$, $\xi = 0.0002$, $m=1$, and $U(T) = B$.

4.3 Behavior of the hadron volume fraction

During the phase transition, the temperature, pressure, enthalpy and entropy are conserved and the quark matter density decreases from ρ_Q to ρ_H , the hadron matter density. During this transition, we have $T_c = 125 \text{ MeV}$, $\rho_Q = 5 \times 10^9 \text{ MeV}^4$, $\rho_H \approx 1.38 \times 10^9 \text{ MeV}^4$, and constant pressure $p_c \approx 4.6 \times 10^8 \text{ MeV}^4$.

In this step, the energy density is described by the volume fraction of matter, namely

$$l62\rho(\tau) = \rho_H h(\tau) + \rho_Q (1 - h(\tau)). \quad (63)$$

At the beginning of the phase transition, we have $\rho(\tau_c) = \rho_Q$, $h(\tau) = 0$, and the whole of the matter in the Universe is described by quarks. However, at the end of the phase transition, the whole of matter is described by hadrons ρ_H , namely $\rho(\tau_h) = \rho_H$, $h(\tau_h) = 1$, and the Universe enters its hadron phase.

The conservation relation implies that

$$l63H = -\frac{r\dot{h}}{D_0 r h + D_1}, \quad (64)$$

where:

$$D_0 = 3\left(1 + \frac{m}{6}\xi\right)$$

$$D_1 = 3\left[1 - \frac{m}{6}\xi\left(\frac{3p_c - \rho_Q - 4\lambda}{\rho_Q + p_c}\right)\right],$$

and r is defined as

$$r = \frac{\rho_H - \rho_Q}{p_c + \rho_Q}.$$

The scale factor is easily obtained as a function of hadron volume fraction

$$l64a(T) = a_c (D_0 r h + D_1)^{-1/D_0}. \quad (65)$$

We find out that the Friedmann equation produces the evolution of the hadron fraction during the quark-hadron phase transition

$$l65\dot{h} = -\frac{D_0 r h + D_1}{r\sqrt{6(6 - (N\xi a\xi)^2)}} \left\{ \left[\rho_Q + (\rho_H - \rho_Q)h \right] \left(2\lambda + \rho_Q + (\rho_H - \rho_Q)h \right) + 6\left(\frac{m}{4\phi}\right)^2 \left(3p_c - \rho_Q - (\rho_H - \rho_Q)h - 4\lambda \right)^2 \right\}^{1/2}. \quad (66)$$

We presented the numerical results of Eq. (??) in Fig. 5 for $\lambda = 10^9 \text{ MeV}^4$ (solid line),

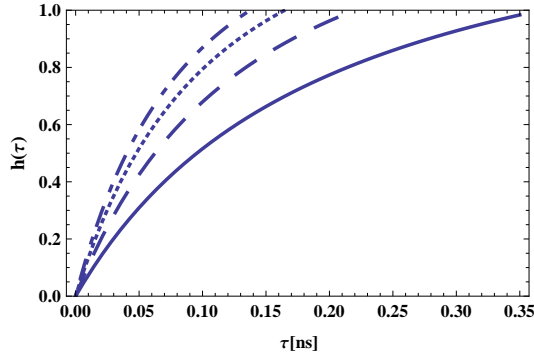


Figure 5: Hadron volume fraction as a function of cosmic time τ for $\lambda = 10^9 \text{ MeV}^4$ (solid line), $\lambda = 5 \times 10^9 \text{ MeV}^4$ (Dashed), $\lambda = 10 \times 10^9 \text{ MeV}^4$ (Dotted line), $\lambda = 15 \times 10^9 \text{ MeV}^4$ (Dashed-dotted line) in the brane world gravity with chameleon scalar field in the bulk. We have set $N = 0.2 \times 10^4$, $\xi = 0.0002$, and $m=1$.

$\lambda = 5 \times 10^9 \text{ MeV}^4$ (Dashed), $\lambda = 10 \times 10^9 \text{ MeV}^4$ (Dotted line), $\lambda = 15 \times 10^9 \text{ MeV}^4$ (Dashed-dotted line). This figure shows that the hadron volume fraction increases with passing time during the quark-hadron phase transition QHPT and it takes about 0.1-0.35 nanosecond. This figure completely agrees with the results of previous figures and by increasing the brane tension the increasing of the hadron volume fraction will be faster. We plot the scale factor of the Universe during the QHPT as a function of hadron volume fraction, h , in Fig. 6. From figure 6 it is seen that during the QHPT the Universe is expanding, although temperature, pressure, enthalpy, and entropy during the phase transition are constant.

4.4 Behavior of temperature in the hadron phase

In this stage of its evolution, the Universe is in a pure hadronic phase, with an equation of state given by

$$l66\rho = 3p = 3a_\pi T^4. \quad (67)$$

Substituting this equation of state into the conservation relation leads to the following differential equation

$$l67H = -\frac{6a_\pi T^3 \dot{T}}{6a_\pi T^4 + m\xi\lambda}. \quad (68)$$

l68 By integrating Eq. (??), one obtains the scale factor as a function of temperature

$$l69a(T) = a_c \left(6a_\pi T^4 + m\xi\lambda \right)^{-1/4}. \quad (69)$$

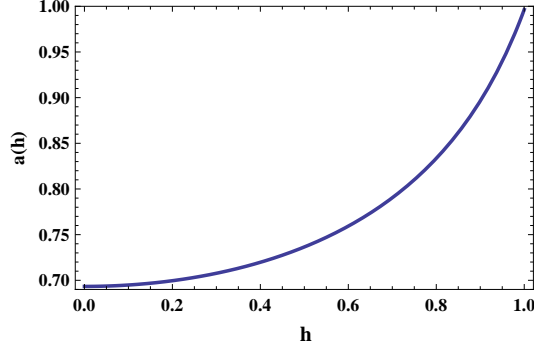


Figure 6: Scale factor as a function of the hadron volume fraction during the QHPT in the brane world gravity with chameleon scalar field in the bulk. We have set $N = 0.2 \times 10^4$, $\xi = 0.0002$, and $m=1$.

Consequently, we get that \dot{T} , in the pure hadronic phase era, is

$$l70\dot{T} = -\frac{6a_\pi T^4 + 2m\xi\lambda}{6a_\pi T^3 \sqrt{6[6 - (N\xi a^\xi)^2]}} \left\{ 3a_\pi T^4 (2\lambda + 3a_\pi T^4) + \frac{6m^2}{\phi_0^2} \right\}^{1/2} \quad (70)$$

We plotted the numerical results of Eq. (??) in Fig. 7 for $\lambda = 10^9$ MeV⁴(solid line),

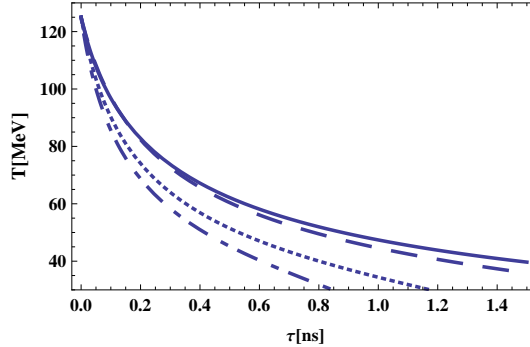


Figure 7: T versus τ in the pure hadronic phase era for $\lambda = 10^9$ MeV⁴(solid line), $\lambda = 5 \times 10^9$ MeV⁴(Dashed), $\lambda = 10 \times 10^9$ MeV⁴(Dotted line), $\lambda = 15 \times 10^9$ MeV⁴(Dashed-dotted line) in the brane world gravity with chameleon scalar field in the bulk. We have set $N = 0.2 \times 10^4$, $\xi = 0.0002$, $m=1$.

$\lambda = 5 \times 10^9$ MeV⁴(Dashed), $\lambda = 10 \times 10^9$ MeV⁴(Dotted line), $\lambda = 15 \times 10^9$ MeV⁴(Dashed-dotted line). This figure shows the effective temperature of the Universe as a function of cosmic time in the brane gravity with chameleon scalar field in the bulk for hadron area. This figure shows that the hadron area is occurred at about 0.8-1.5 nanosecond after the big bang and this fact confirms the results of other areas.

5 Discussion and Conslusion

In this paper we considered the quark-hadron phase transition in brane-world model including a bulk scalar field. The peroblem of quark-hadron phase transition has been in-

investigated in different models of brane-world scenario. Quark-hadron phase transition in RS brane-world model has been performed in [24]. It was shown that the temperature of the early universe in brane-world model is smaller than standard model, and small value of brane tension reduces the quark-gluon plasma temperature and accelerate phase transition to hadron era. In addition, hadron fraction, h , strongly depends on brane tension and small value of brane tension leads to bigger value of h relative to standard model. In [24] the effect of dark radiation also has been considered and it was obtained that large value of dark radiation strongly accelerate the formation of hadron phase. In [65], a Brans-Dicke(BD) brane-world model has been utilized to investigate quark-hadron phase transition. The temperature of the early universe is smaller than RS brane-world model. Quark-gluon plasma temperature is reduced by increasing the BD coupling constant, ω . The hadron fraction depends on the BD coupling as well in which large value of coupling constant leads to higher h and finally smaller time interval is necessary for phase transition. We also considered variation of temperature by using crossover approach in high and low temperature. In smooth crossover regime, where lattice QCD for high temperature was used, there is a smooth slope relative to first order phase transition and for low temperature where HRG is used, the slope is steep relative to first order phase transition.

The framework which has been utilized in our work is a generalized form of [65] in which the bulk scalar field has a non-minimal coupling to matter as well as a non-minimal coupling to geometry. Many authors have tried to add some extra terms to the energy momentum for obtaining a brane world model with the bulk-brane energy transfer, but in our model the energy transfer is caused by interaction between the scalar field in the bulk and the matter field on the brane. In fact, due to this bulk-brane energy transfer and modified Friedmann equation the rate of expansion of the Universe is increased in the early time. In this work we studied the QCD phase transition with two different mechanisms; smooth crossover approach and first-order phase transition formalism. We took into consideration the dynamical evolution of physical quantities such as, energy density, effective temperature and scale factor, before, during and after the phase transition. Our analysis shows that, the quark-hadron phase transition has taken place at about nanosecond after the big bang. First we studied the QCD phase transition utilizing smooth crossover formalism in two regime of high and low temperature. Equation of state is an important relation in quark hadron phase transition. A lattice data can be utilized for trace anomaly to construct an appropriate EoS. In high temperature regime, namely $T \geq 250\text{MeV}$ the trace anomaly can be computed accurately which shows a radiation like behavior; however in low temperature regime $T \leq 150\text{MeV}$ the trace anomaly is affected by large discretization effect. So HRG model is used to build a realistic EoS in this regime. Considering the results in detailed indicate some differences. Our analysis shows that in the high temperature region of the smooth crossover the effective temperature of the QGP of the Universe is in the interval $\text{MeV } 250 \leq T \leq 700$ and QGP has occurred (finished) at about 0.2-0.3 nanosecond after the big bang. And in the low temperature region of the smooth crossover the effective temperature of the QGP of the Universe is in the interval $\text{MeV } 50 \leq T \leq 180$ and QGP has occurred (finished) at about 15 nanosecond after the big bang. Note that this result is about 10^{-3} times earlier than the prediction of other works [58, 60].

Then we studied phase transition using the first order phase transition formalism where we have three stages; before (QGP), during and after (hadron) phase transition. We find out that for two different models; general $U(T)$ and bag model with $U(T) = \text{constant}$, the effective temperature of the QGP is in the interval $\text{MeV } 125 \leq T \leq 200$ at about 0.05-0.2 nanosecond after the big bang. It is notable that the QGP in the bag model took place a

little faster than general $U(T)$. Furthermore our analysis shows that the phase transition in the first order phase transition formalism took about 0.35 nanosecond and during the quark-hadron phase transition (QHPT) the Universe is expanded although the temperature, pressure, enthalpy and entropy of the system are constant. Finally we examine that the hadronic area takes place in the interval $\text{MeV } 30 \leq T \leq 125 \text{ MeV}$ at about 0.8-1.4 nanosecond after the big bang. The results of this case was solve numerically and depicted for different values of brane tension. It was realized that large value of brane tension significantly reduce the temperature of early universe and accelerate the phase transition. The relation between temperature and brane tension in this work is unlike [24]. It seems that the mechanism which bring such a result is the coupling of scalar field to other components. At last our study shows that the general behavior of the effective temperature of the Universe is similar in the first order phase transition formalism and smooth crossover approach.

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